

# Vortex drag in a Thin-film Giaever transformer

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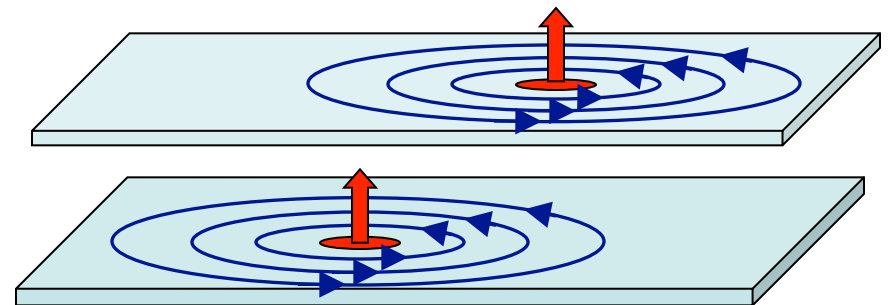
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Matthew Fisher (station Q)

T. Senthil (MIT)



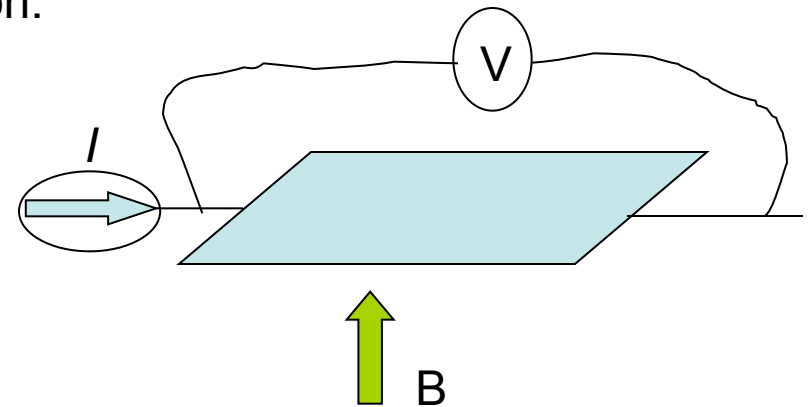
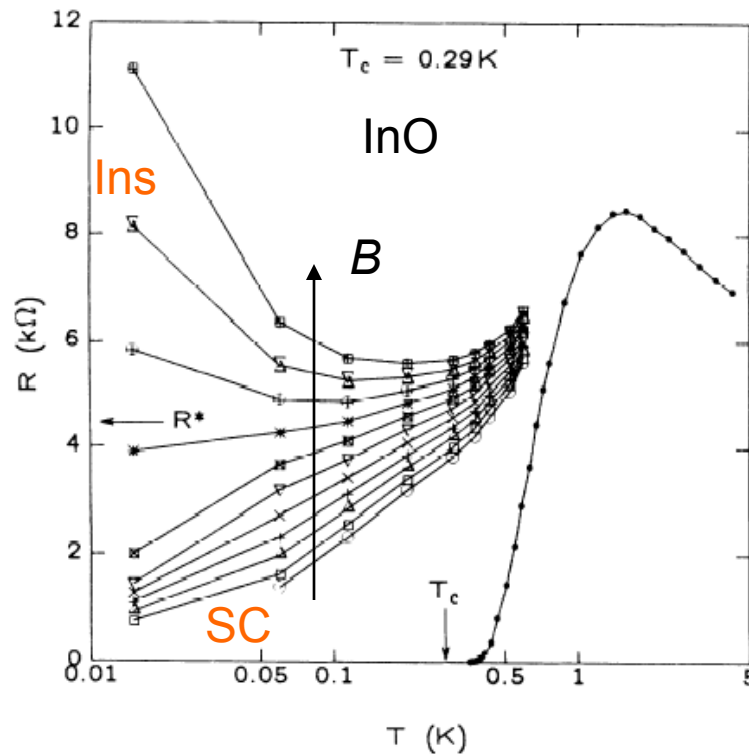
# Outline

- Experimental background – SC-metal-insulator in InO, TiN, Ta and MoGe.
- Two paradigms:
  - Vortex condensation: Vortex metal theory.
  - Percolation paradigm
- Thin film Giaever transformer – amorphous thin-film bilayer.
- Predictions for the no-tunneling regime of a thin-film bilayer
- Conclusions

# Quantum vortex physics

## SC-insulator transition

- Thin films:  $B$  tunes a SC-Insulator transition.



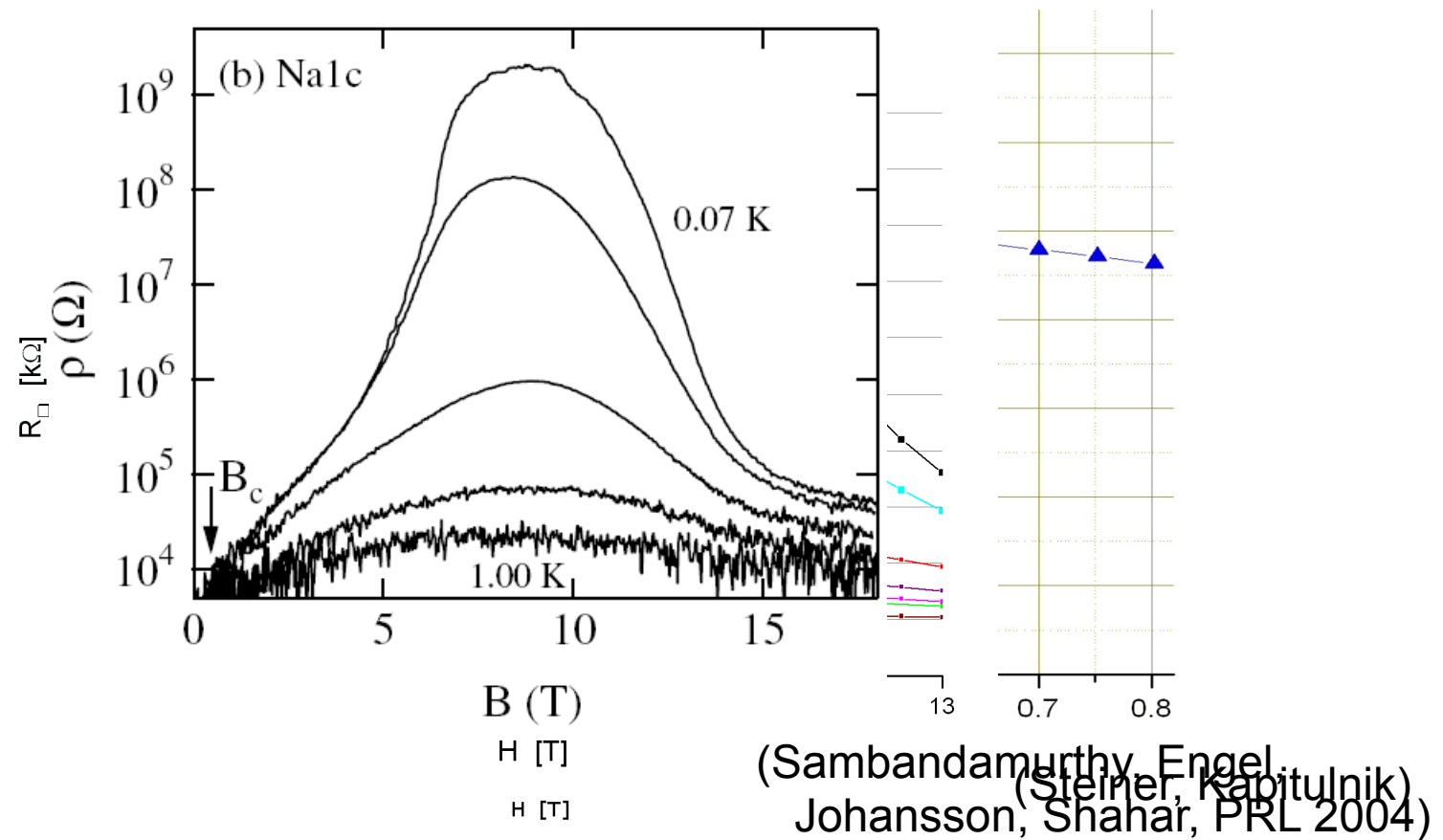
(Hebard, Paalanen, PRL 1990)

FIG. 1. Logarithmic plots of the resistance transitions in zero field ( $\bullet$ ) and nonzero field (open symbols) for a film with  $T_c = 0.29$  K. The isomagnetic lines range from  $B = 4$  kG ( $\circ$ ) to  $B = 6$  kG ( $\square$ ) in 0.2-kG steps. The horizontal and vertical arrows identify  $R^*$  and  $T_c$ , respectively.

# Observation of Superconductor-insulator transition

- Thin amorphous films: B tunes a SC-Insulator transition.

InO:



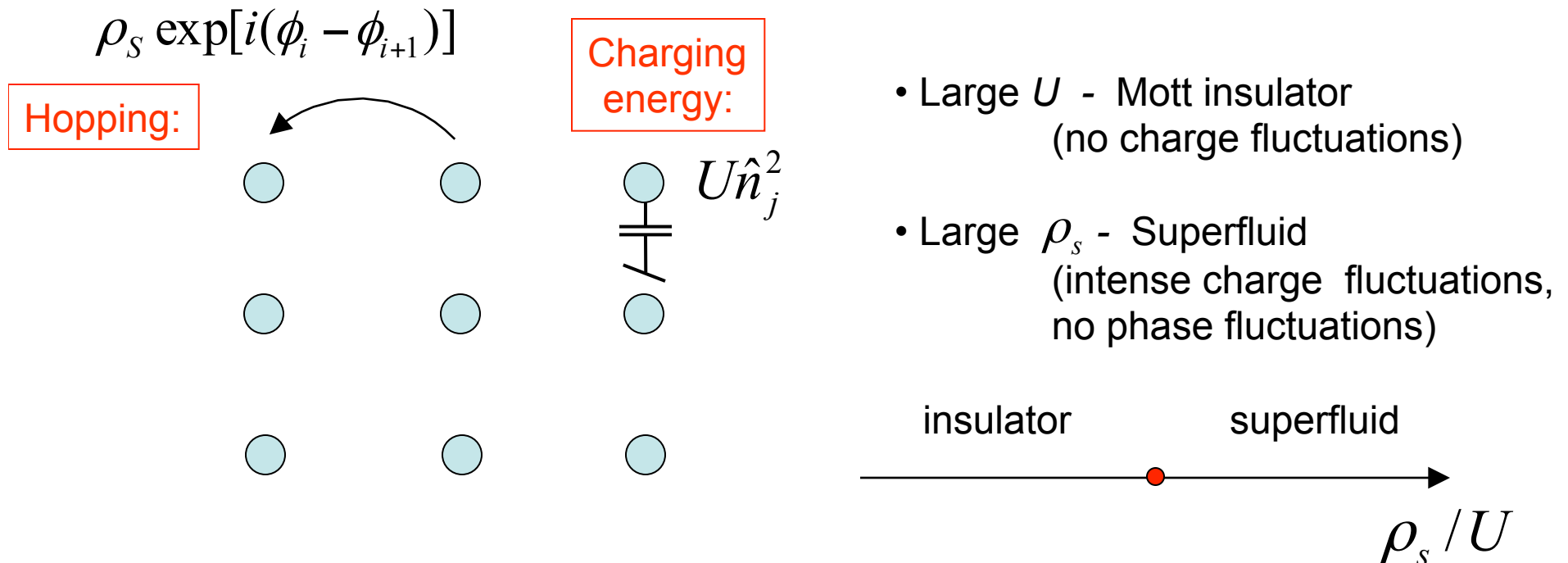
- Saturation as  $T \rightarrow 0$
- Insulating peak different from sample to sample, scaling different – log, activated.

# *Vortex Paradigm*

## X-Y model for superconducting film: Cooper pairs as Bosons

- When the superconducting order is strong – ignore electronic excitations.
- Standard model for bosonic SF-Ins transition – “Bose-Hubbard model”:

$$H = \sum_i \left\{ - \sum_j \rho_s \cos(\nabla_j \phi_i) + U \hat{n}_i^2 \right\} \quad [\hat{n}, \phi] = -i$$



# Vortex description of the SF-insulator transition

(Fisher, 1990)

- Vortex hopping: (result of charging effects)

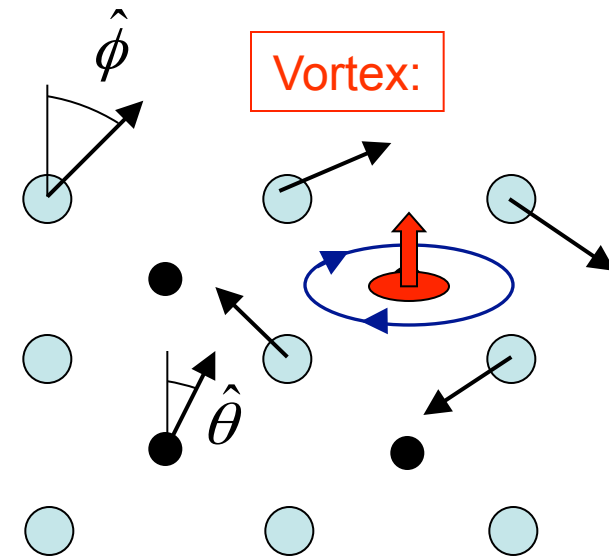
$$-t_V \cos(\nabla_j \hat{\theta}_i)$$

$$[\hat{n}_V, \theta] = -i$$

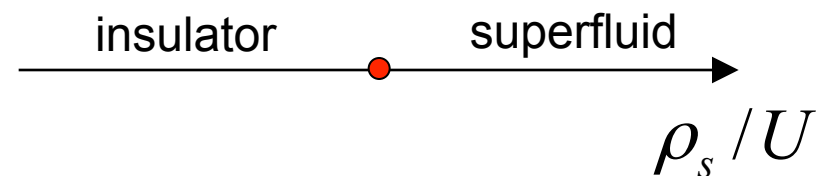
- Vortex-vortex interactions:

$$\frac{1}{2} \rho_s \sum_{i,j} n_{Vi} \cdot n_{Vj} \cdot \ln |\vec{x}_i - \vec{x}_j|$$

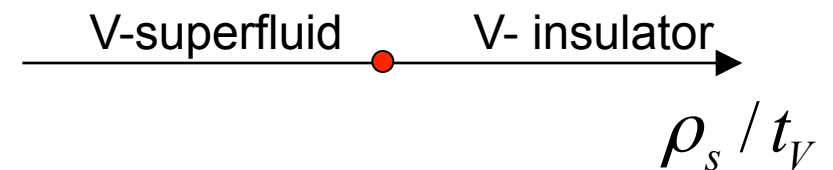
**Condensed vortices  
= insulating CP's**



Cooper-pairs:



Vortices:



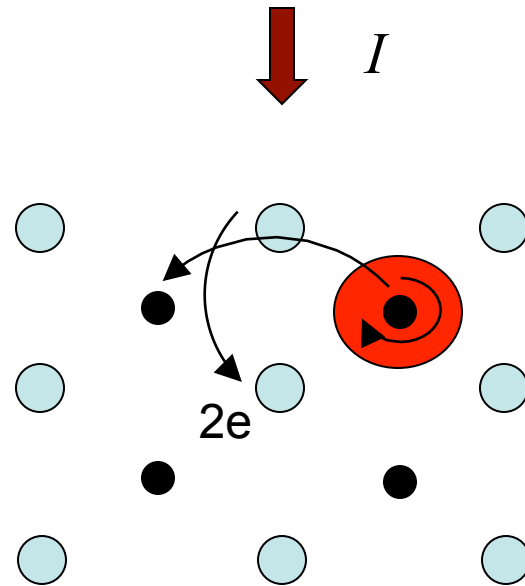
## Universal (?) resistance at SF-insulator transition

Assume that vortices and Cooper-pairs are self dual at transition point.

- Current due to CP hopping:  $I = \frac{2e}{\tau}$
- EMF due to vortex hopping:

$$\frac{\hbar}{2e} \Delta \dot{\phi} = \Delta V \quad \rightarrow \quad \Delta V = \frac{\hbar}{2e} \frac{2\pi}{\tau}$$

- Resistance:  $R = \frac{V}{I} = \frac{2\pi\hbar}{2e\tau} \bigg/ \frac{2e}{\tau} = \frac{h}{4e^2} = 6.5k\Omega$



In reality superconducting films are not self dual:

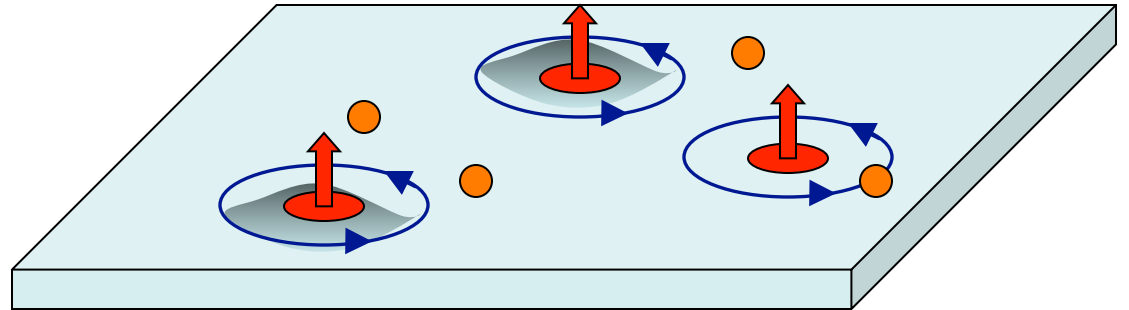
- vortices interact logarithmically, Cooper-pairs interact at most with power law.
- Samples are very disordered and the disorder is different for cooper-pairs and vortices.



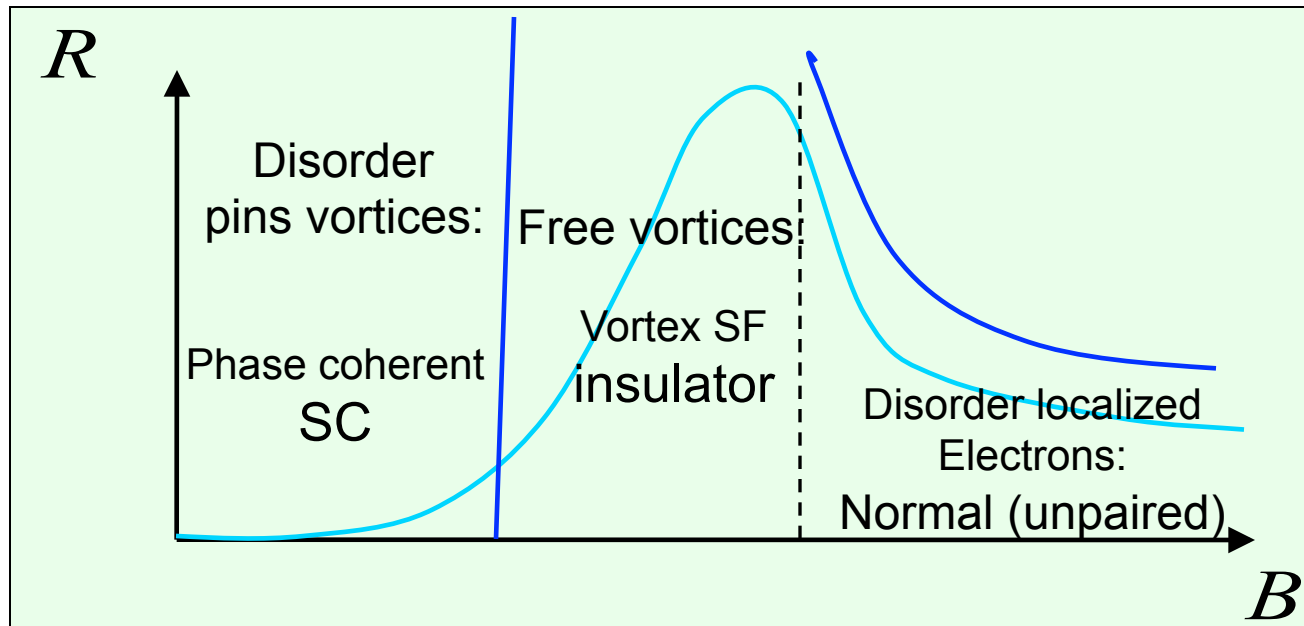
# Magnetically tuned Superconductor-insulator transition

- Net vortex density:

$$\frac{h}{2e} \langle \hat{n}_V \rangle = B$$



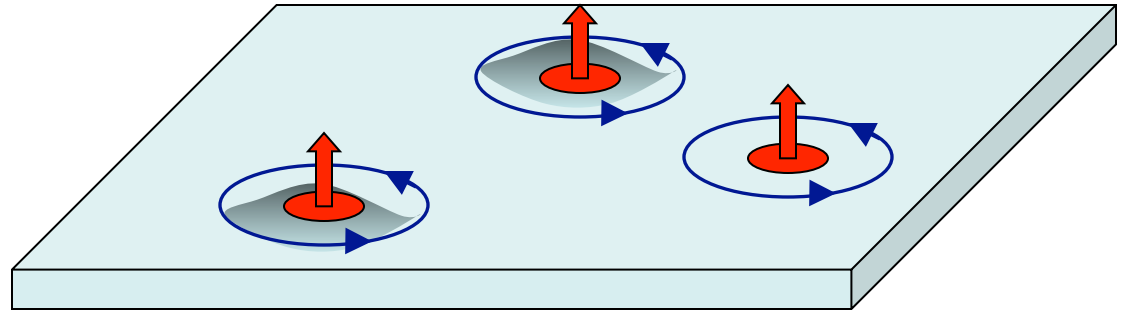
- Disorder pins vortices for small field – **superconducting phase**.
- Large fields some free vortices appear and condense – **insulating phase**.
- Larger fields superconductivity is destroyed – **normal (unpaired) phase**.



# Magnetically tuned Superconductor-insulator transition

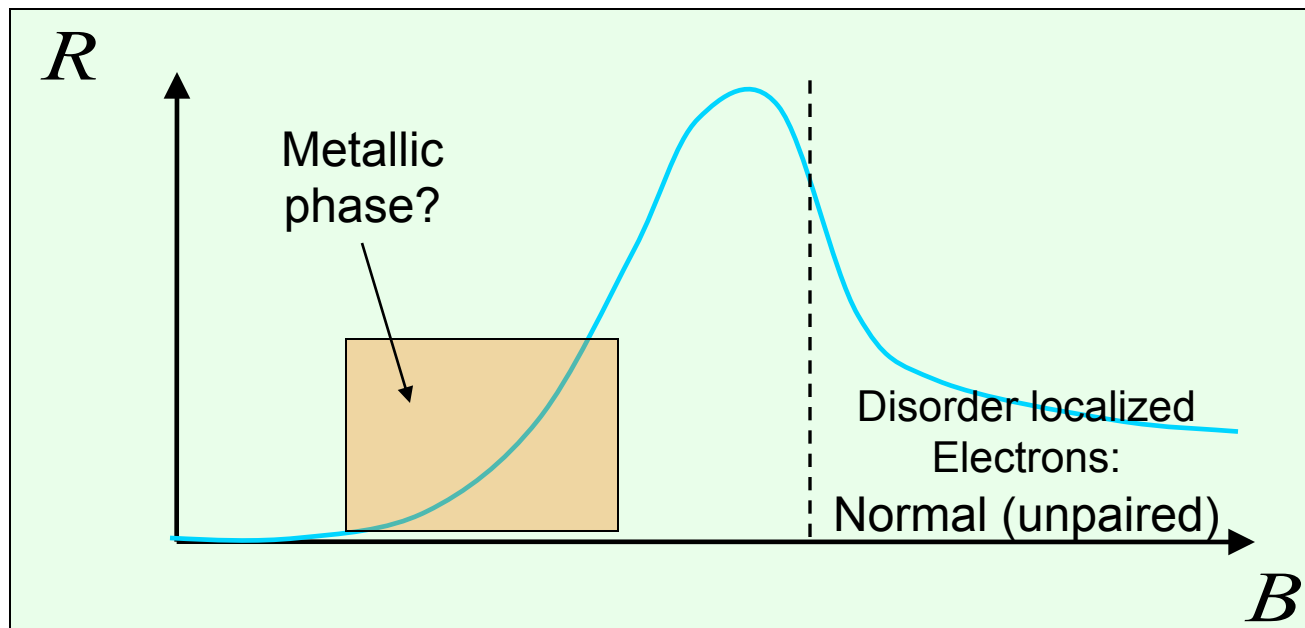
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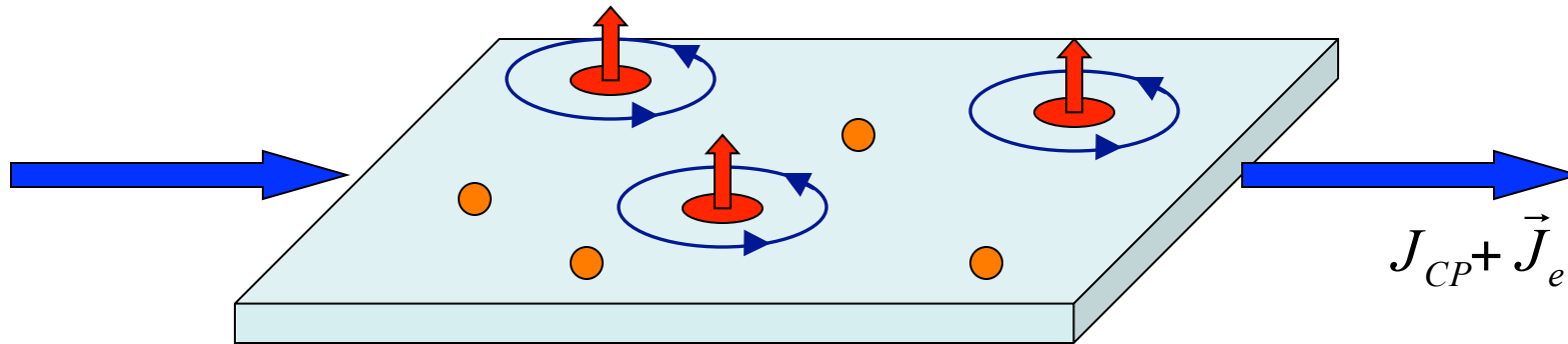
## Problems

- Saturation of the resistance – ‘metallic phase’
- Non-universal insulating peak – completely different depending on disorder.

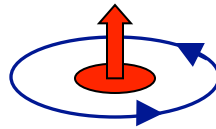


# Two-fluid model for the SC-Metal-Insulator transition

(Galitski, Refael, Fisher, Senthil, 2005)



**Uncondensed vortices:  
Cooper-pair channel**



Finite conductivity:

$$\vec{j}_V = \sigma_V \vec{F}_V \Rightarrow \boxed{\frac{1}{\sigma_V} \vec{E} = \vec{J}_{CP}}$$

$$\vec{F}_V = \hat{z} \times \vec{J}_{CP}$$

$$\hat{z} \times \vec{E} = \vec{j}_V$$

**Disorder induced Gapless QP's  
(electron channel)**

(delocalized core states?)

$$\boxed{\vec{J}_e = \sigma_e \vec{E}}$$

**Two channels in parallel:**

$$\boxed{\vec{J} = (\sigma_e + \sigma_V^{-1}) \vec{E}}$$

# Transport properties of the vortex-metal

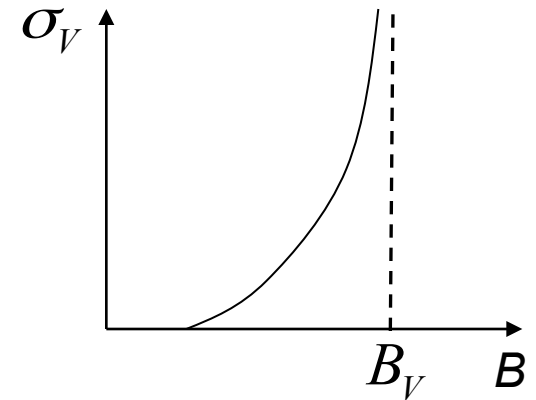
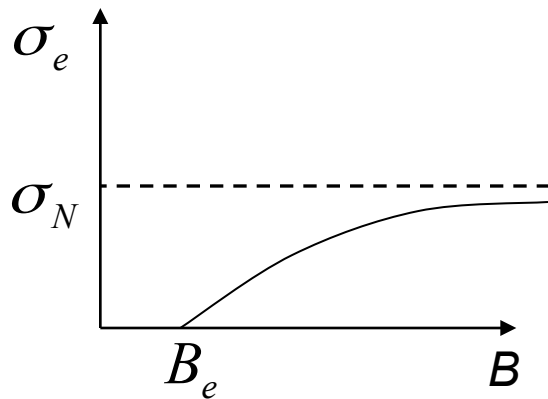
Effective conductivity:

$$\sigma_{eff} = \sigma_e + \sigma_V^{-1}$$

• **Assume:**

-  $\sigma_e$  grows from zero to  $\sigma_N$ .

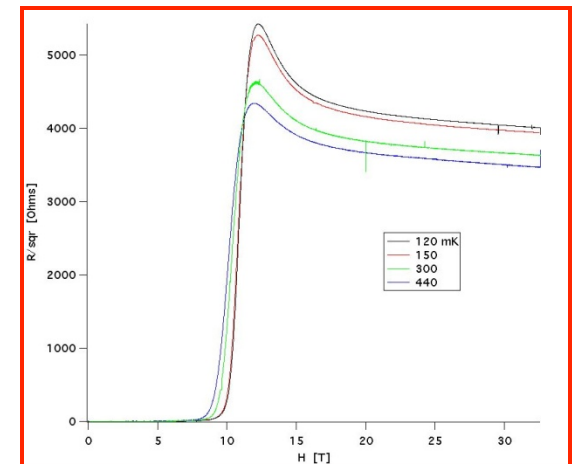
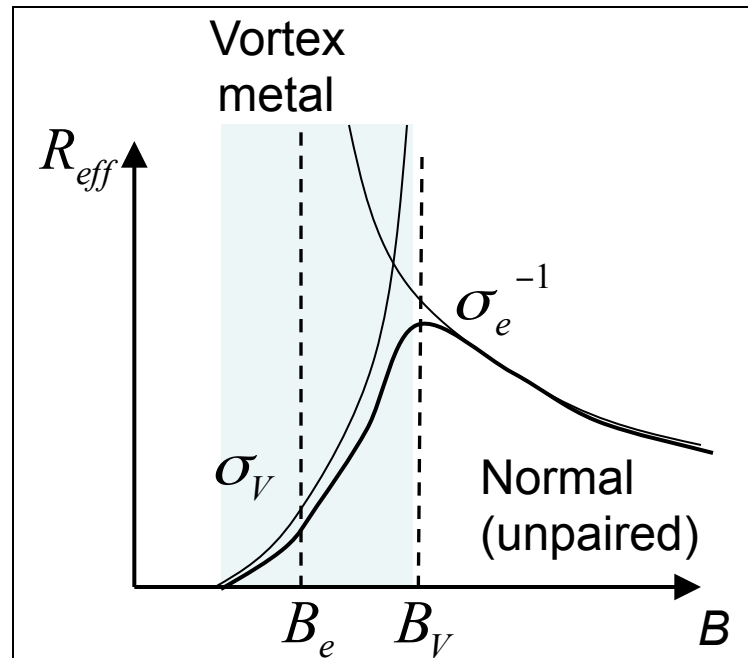
-  $\sigma_V$  grows from zero to infinity.



**Weak insulators:**

Ta, MoGe  
Weak InO

$$B_e < B_V$$



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Effective conductivity:

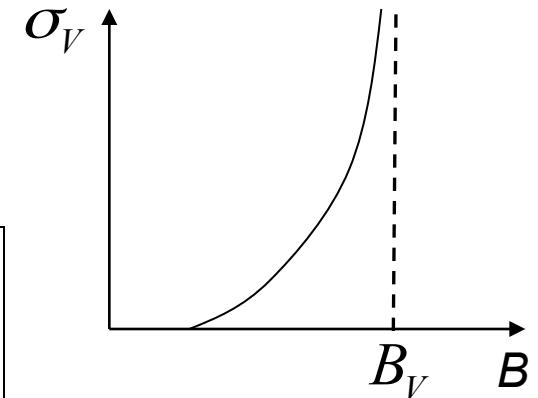
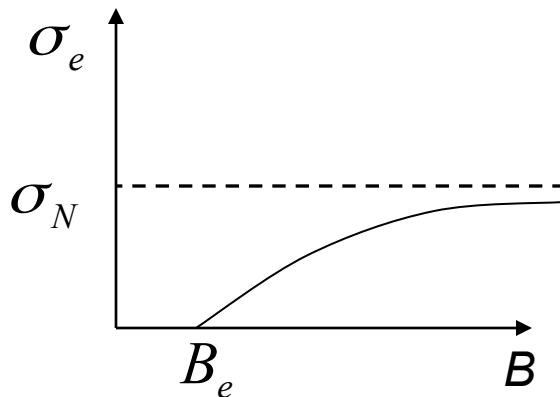
$$\sigma_{eff} = \sigma_e + \sigma_V^{-1}$$

*Chargless spinons contribute to conductivity!*

• Assume:

-  $\sigma_e$  grows from zero to  $\sigma_N$ .

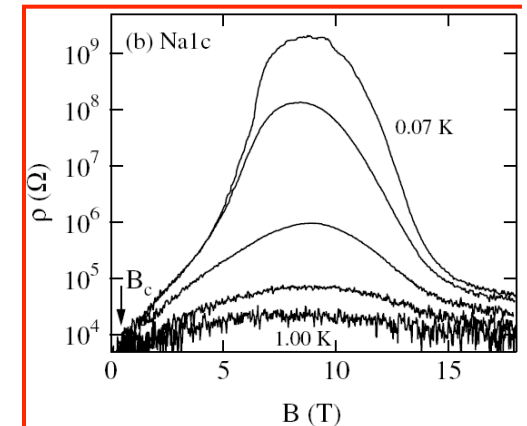
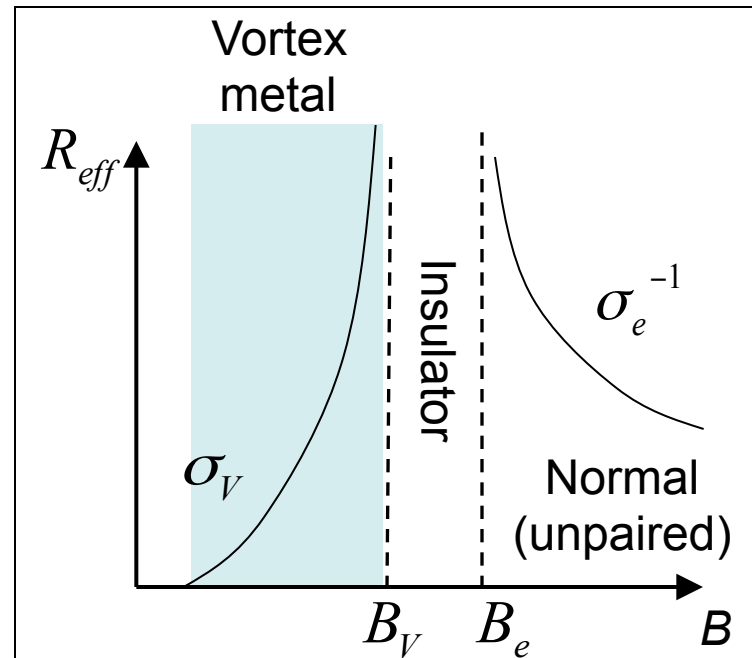
-  $\sigma_V$  grows from zero to infinity.



**Strong insulators:**

TiN, InO

$$B_e > B_V$$

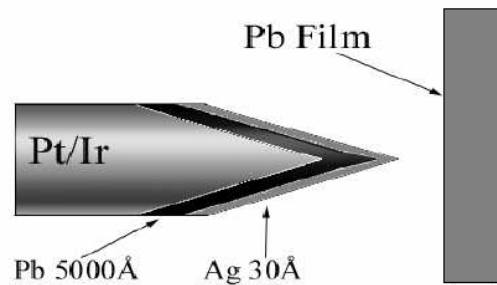


## More physical properties of the vortex metal

### *Cooper pair tunneling*

- A superconducting STM can tunnel Cooper pairs to the film:

$$G = G_{2e} + G_{CP}$$



(Naaman, Tyzer, Dynes, 2001).

# More physical properties of the vortex metal

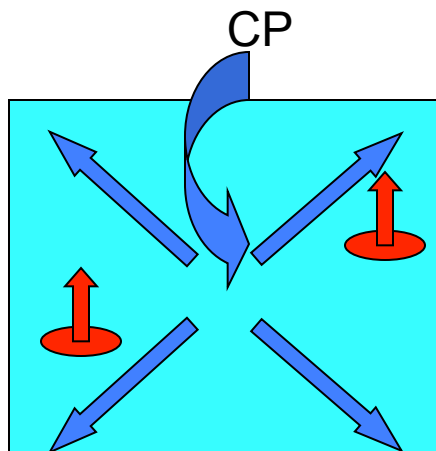
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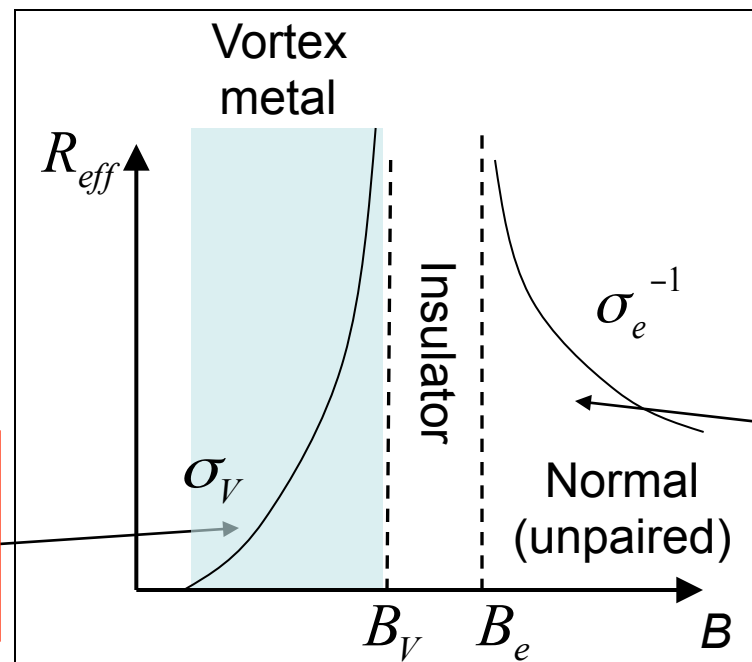
Vortex metal phase:

$$G_{CP} \sim \frac{1}{T^2} \exp(-\sigma_V \ln^2 T)$$



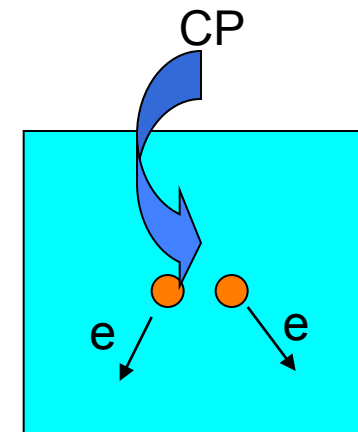
$$G \approx G_{CP}$$

strongly T dependent



Normal phase:

$$G_{2e} \sim \sigma_e^2$$



$$G \approx G_{2e} \sim 1/\ln^2 T$$

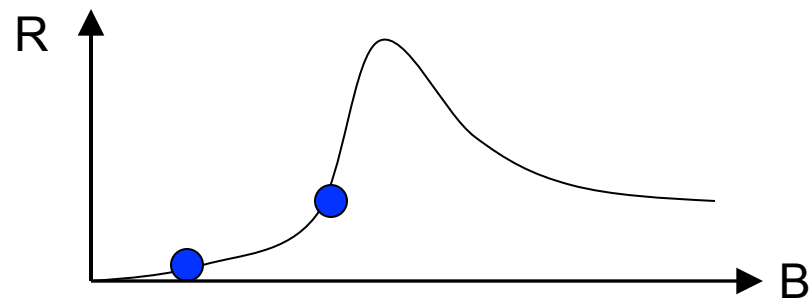
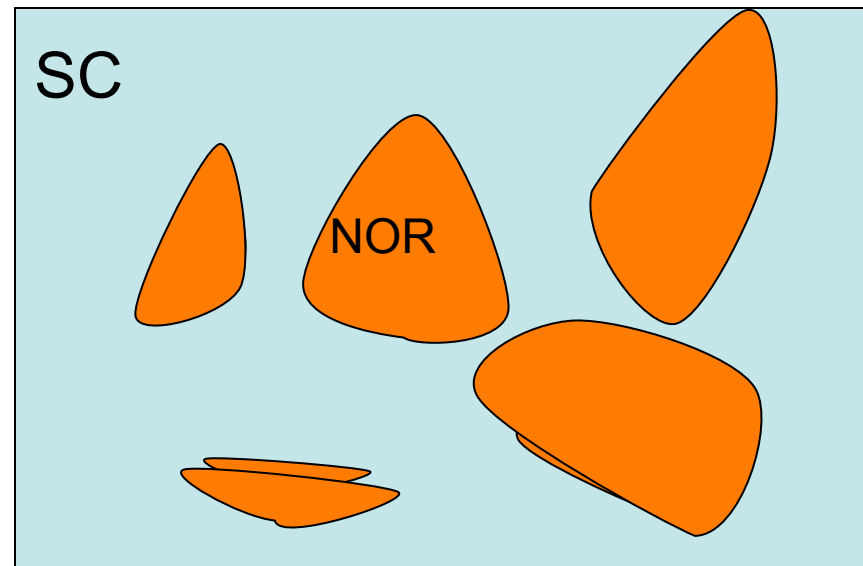
# *Percolation Paradigm*

(Trivedi,  
**Dubi, Meir, Avishai,**  
Spivak, Kivelson,  
et al.)



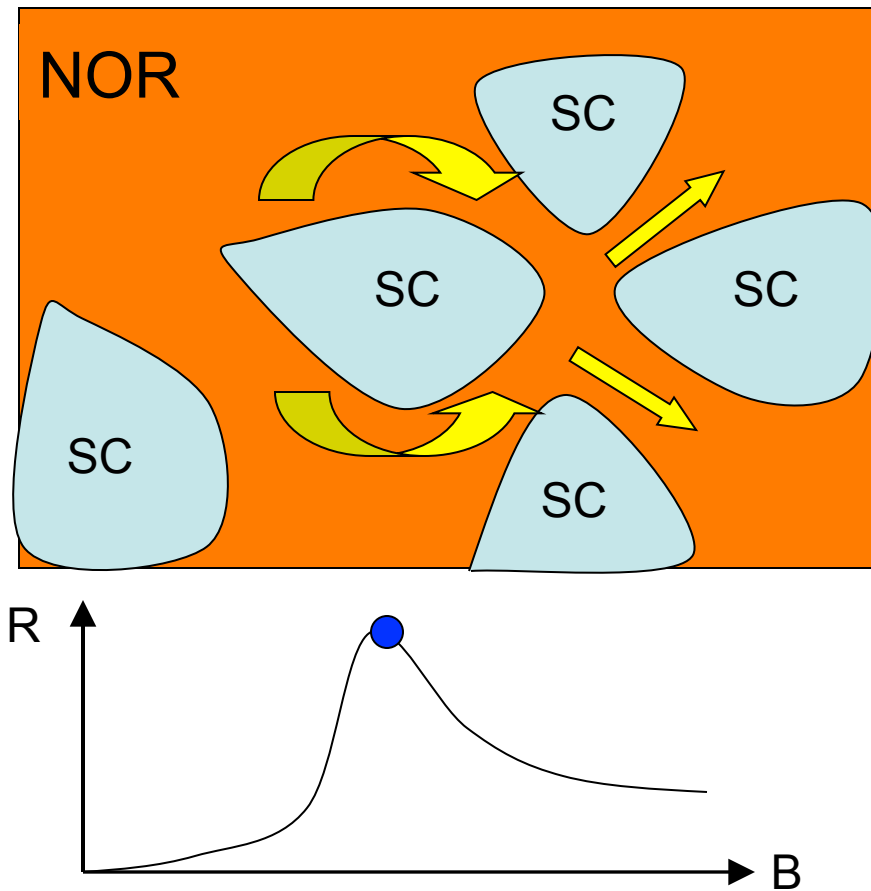
## Pardigm II: superconducting vs. Normal regions percolation

- Strong disorder breaks the film into superconducting and normal regions.



## Pardigm II: superconducting vs. Normal regions percolation

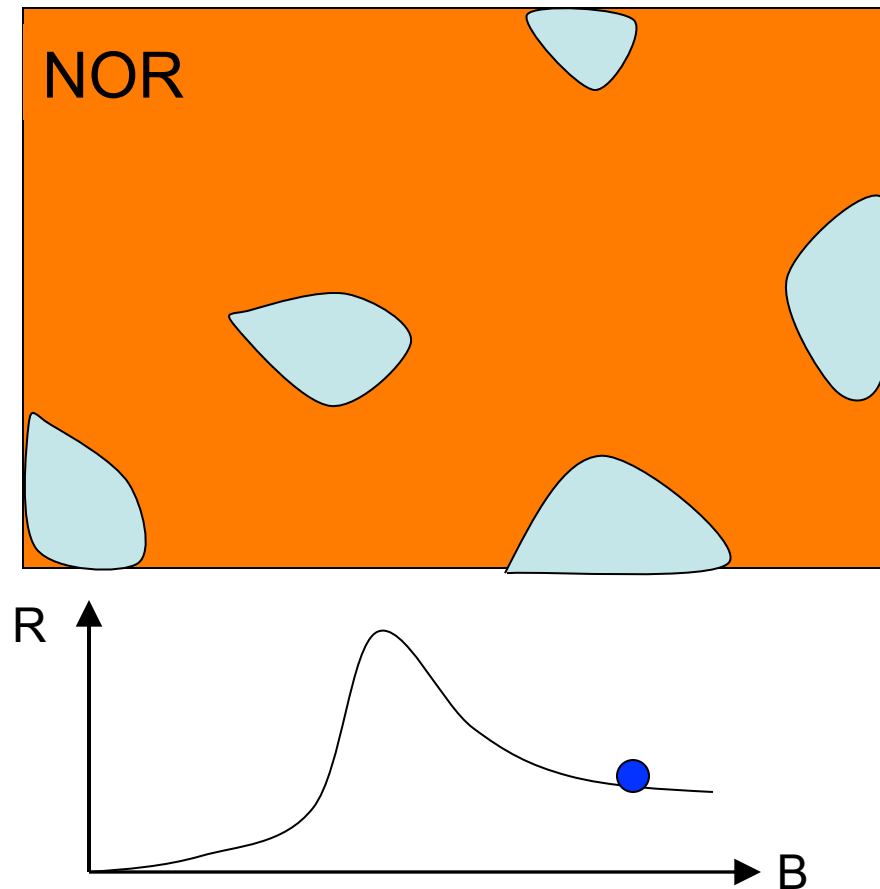
- Strong disorder breaks the film into superconducting and normal regions.



- Near percolation – thin channels of the disorder-localized normal phase.

## Pardigm II: superconducting vs. Normal regions percolation

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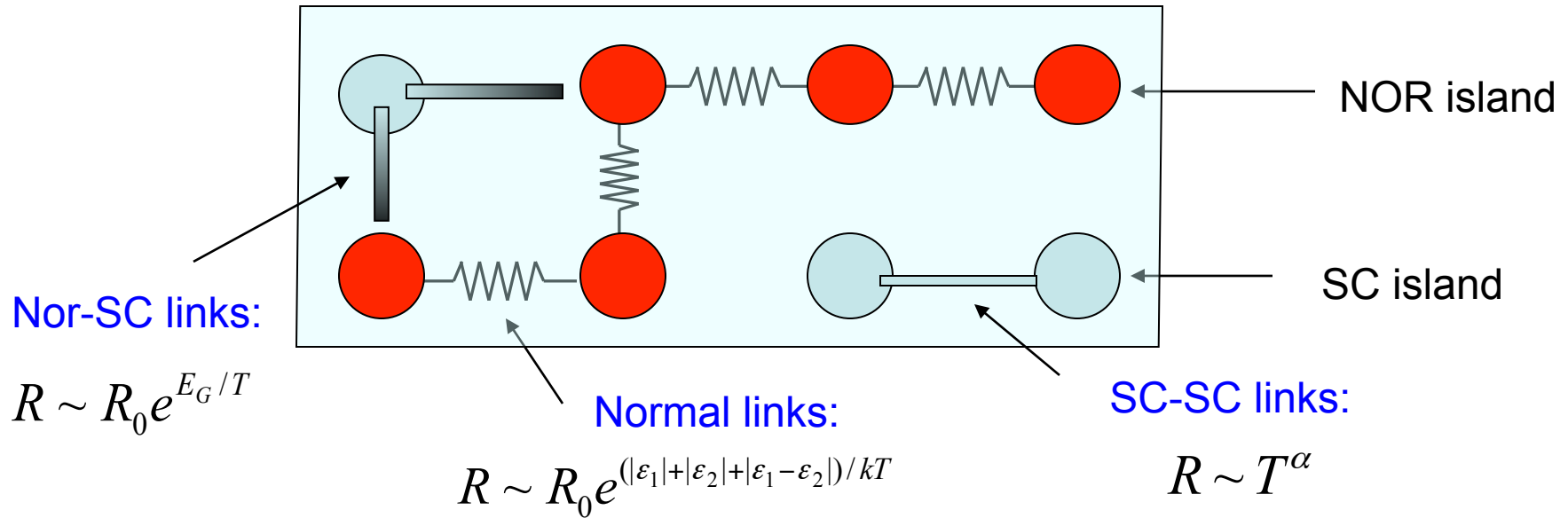


- Near percolation – thin channels of the disorder-localized normal phase.
- Far from percolation – disordered localized normal electrons.

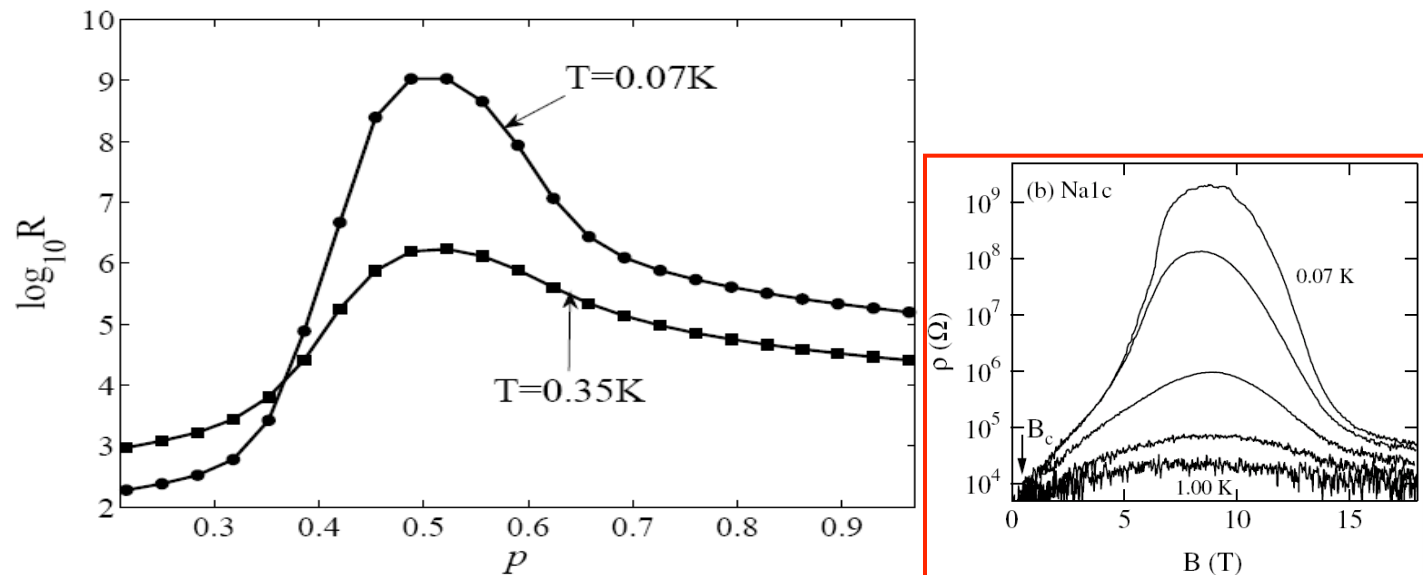
# Magneto-resistance curves in the percolation picture

(Dubi, Meir, Avishai, 2006)

- Simulate film as a resistor network:



**Resulting MR:**

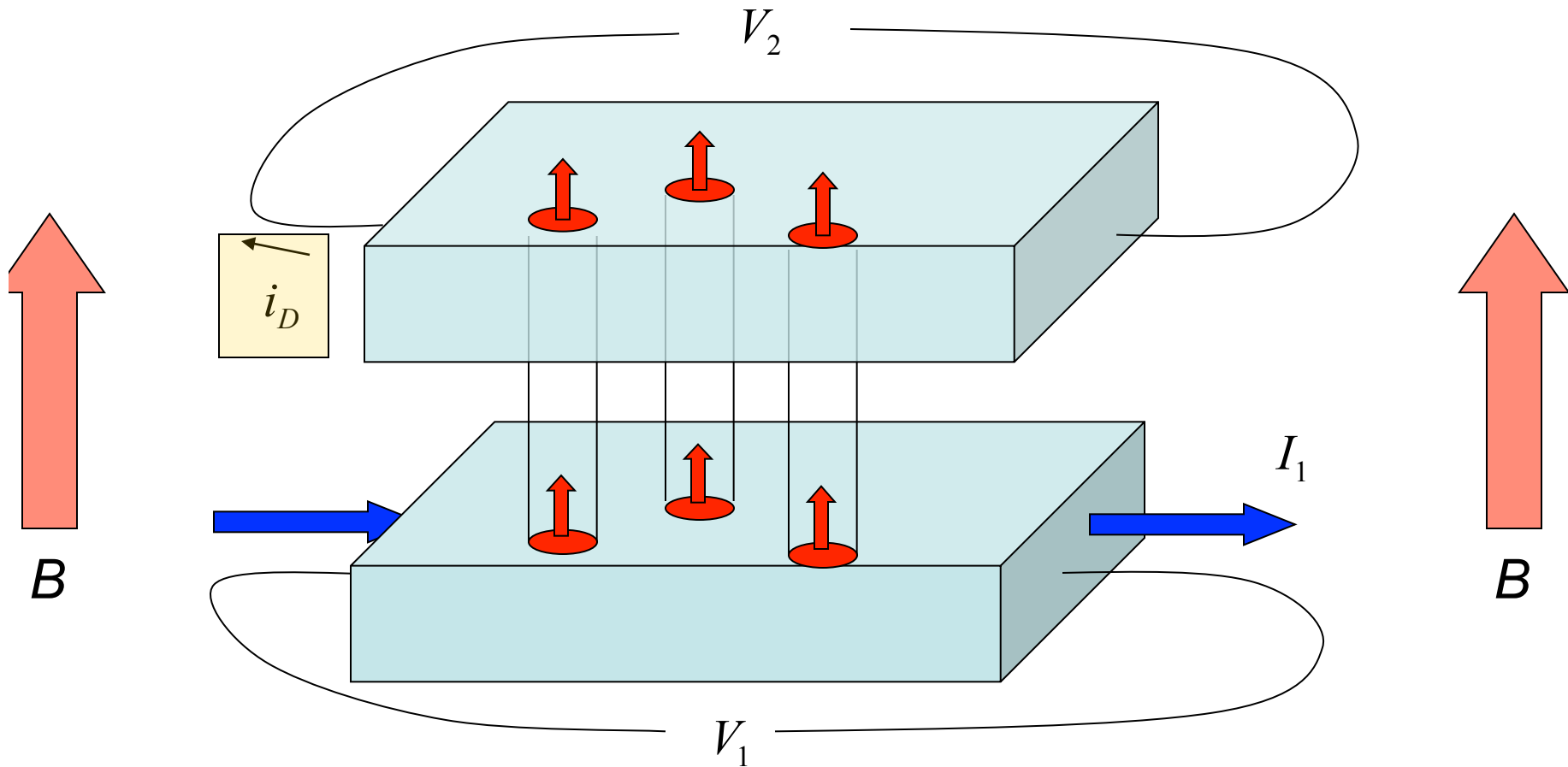


*Drag in a bilayer system*

# Giaever transformer – Vortex drag

(Giaever, 1965)

Two type-II bulk superconductors:



Vortices tightly bound:

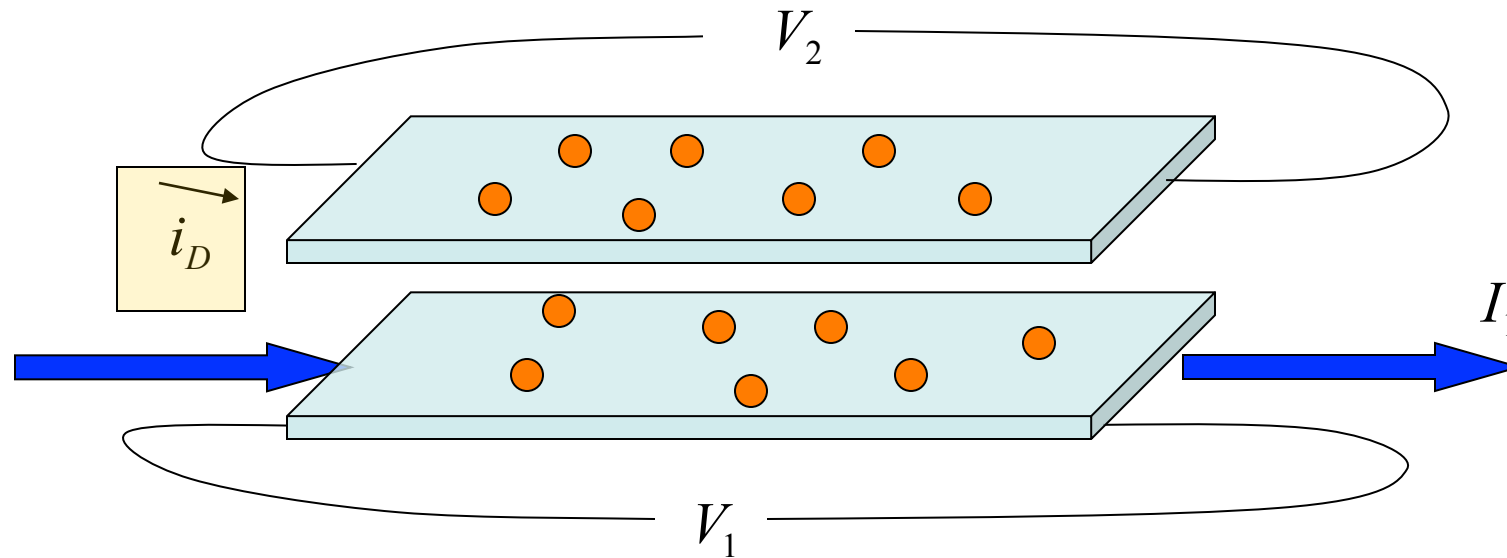
$$V_2 = V_1$$

$$R_D = \frac{V_2}{I_1}$$

## 2DEG bilayers – Coulomb drag

Two thin electron gases:

$$R_D = \frac{V_2}{I_1}$$

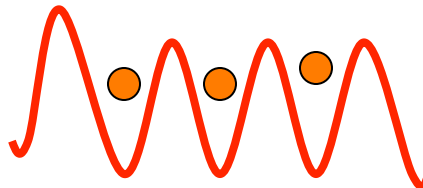
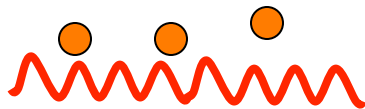


- Coulomb force creates friction between the layers.

- Inversely proportional to density squared:

$$R_D \propto \frac{1}{n_e^2}$$

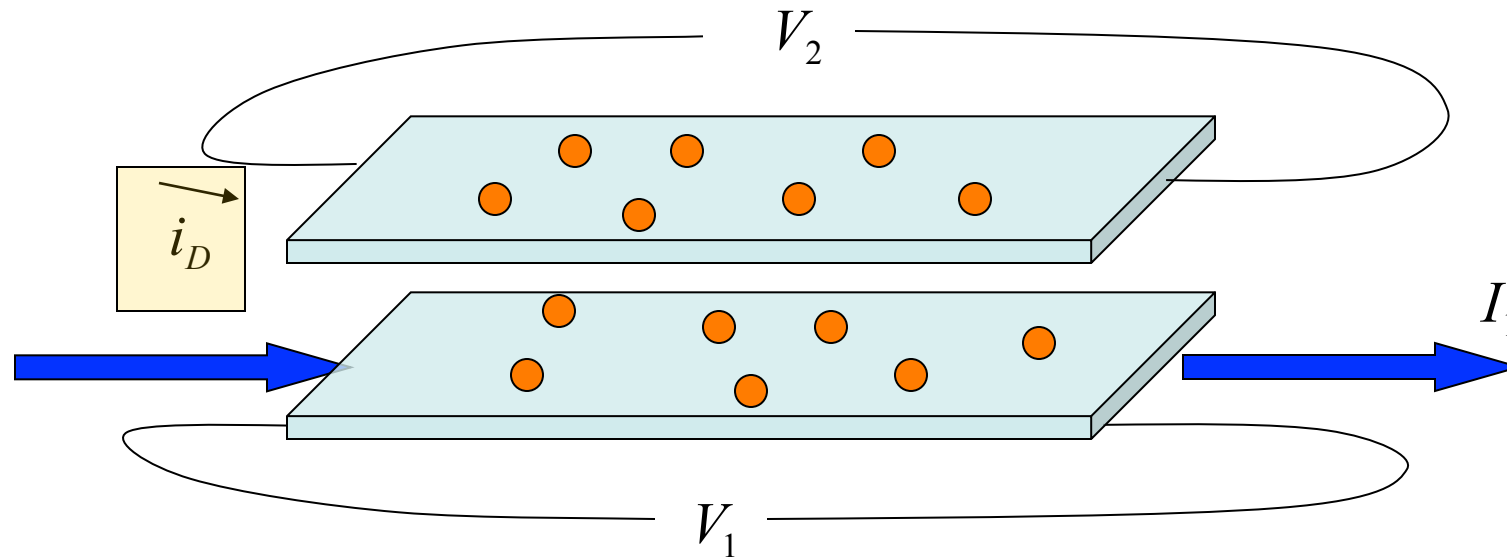
- Opposite sign to Giaever's vortex drag.



## 2DEG bilayers – Coulomb drag

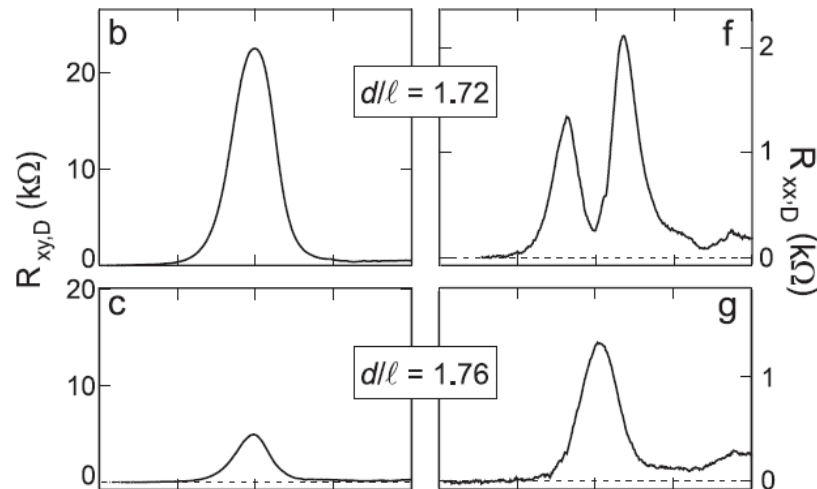
Two thin electron gases:

$$R_D = \frac{V_2}{I_1}$$



Example:

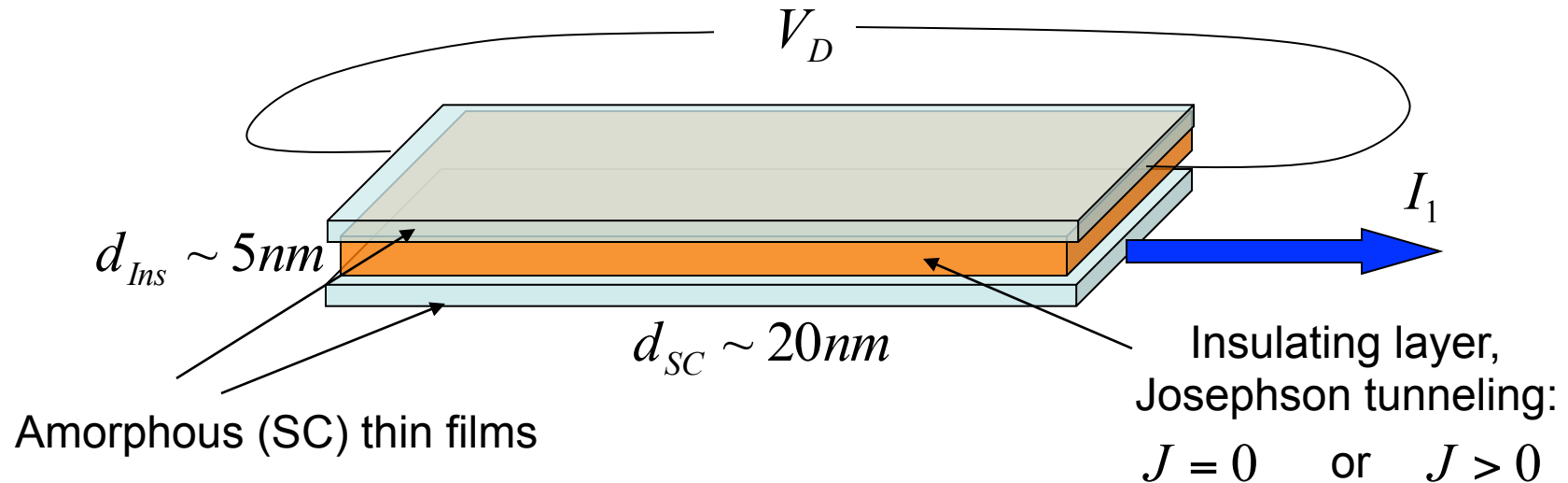
$\nu_T = 1$  “Excitonic condensate”



(Kellogg, Eisenstein, Pfeiffer, West, 2002)



## Thin film Giaever transformer



### Percolation paradigm

- Drag is due to coulomb interaction.
- Electron density:

$$n_{2d} \sim d_{SC} \cdot 10^{20} cm^{-3} \rightarrow 2 \cdot 10^{14} cm^{-2}$$

( QH bilayers:  $n_{2d} \sim 5 \cdot 10^{10} cm^{-2}$  )

**Drag suppressed**

?

### Vortex condensation paradigm

- Drag is due to inductive current interactions, and Josephson coupling.
- Vortex density:

$$n_V \sim \frac{B}{\phi_0} \sim (10^{11} \cdot B_{[T]}) cm^{-2}$$

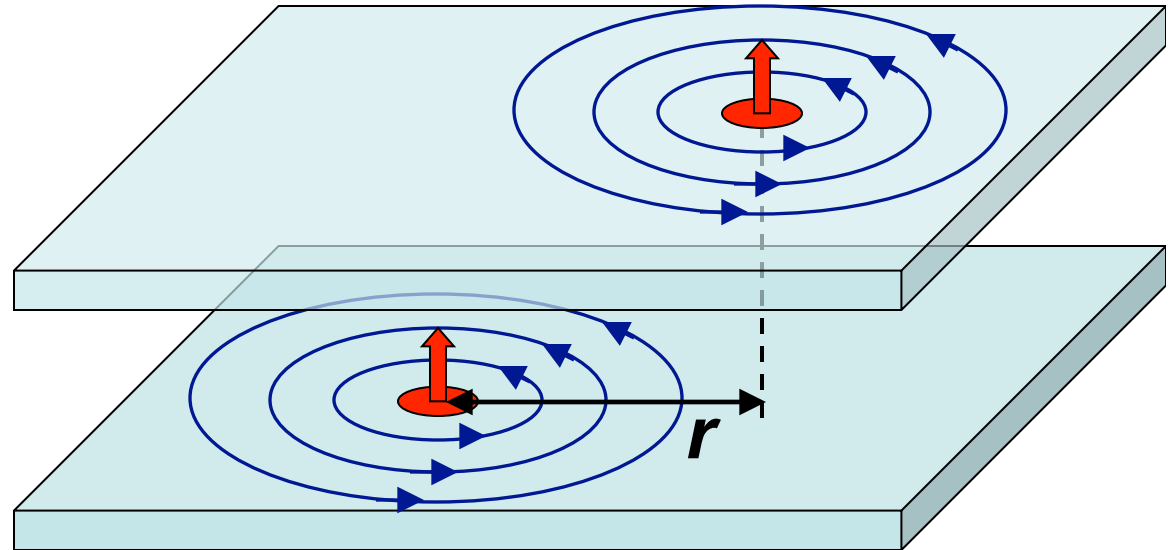
**Significant Drag**

# Vortex drag in thin films bilayers: interlayer interaction

- Vortex current suppressed.

e.g., Pearl penetration length:

$$\lambda_{eff} = \frac{2\lambda_L^2}{d_{SC}}$$



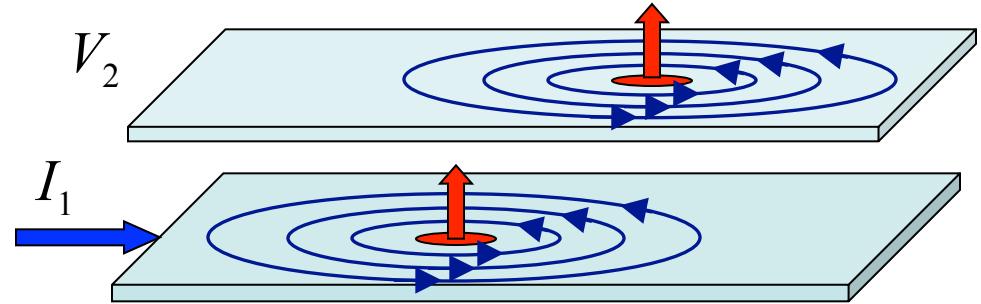
- Vortex attraction=interlayer induction.  
Also suppressed due to thinness.

$$U_{inter}(q) \approx \underbrace{\frac{\phi_0^2}{2\pi\lambda_{eff}^2}}_{\propto d_{SC}^2} \frac{e^{-qa}}{q[(q + \lambda_{eff}^{-1})^2 - e^{-2qa} / \lambda_{eff}^2]}$$

$$\begin{aligned} &\sim \frac{\phi_0^2}{4\pi\lambda_{eff}} \ln(r), \quad r > \lambda_{eff} \\ &\sim \frac{\phi_0^2}{4\lambda_{eff}^2} \frac{r^2}{a}, \quad r < a \end{aligned}$$

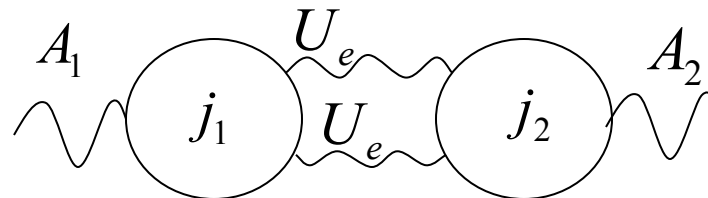
# Vortex drag in thin films within vortex metal theory

- $R_D = \frac{V_2}{I_1}$



- Perturbatively:

$$R_D = G_V^{drag} \sim [j_1, j_2]$$



(Kamenev, Oreg)

- Expect:  $G_V^{drag} \propto \frac{\partial \sigma_{V1}}{\partial n_{V1}} \cdot \frac{\sigma_{V2}}{n_{V2}} \Rightarrow \boxed{\propto \frac{\partial R_1}{\partial B} \cdot \frac{\partial R_2}{\partial B}}$  **Drag generically proportional to MR slope.**

(Following von Oppen, Simon, Stern, PRL 2001)

- Answer:

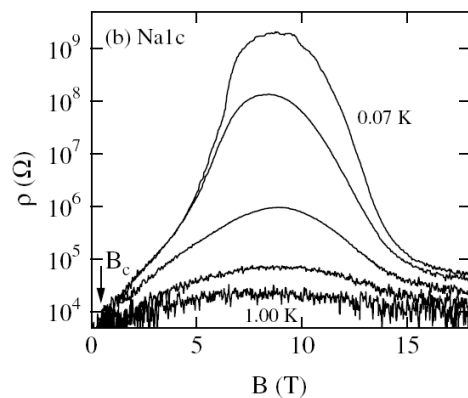
$$R_D = G_V^{drag} = \frac{e^2 \phi_0^2}{8\pi^4 T} \frac{\partial R_1}{\partial B} \frac{\partial R_2}{\partial B} \int d\omega \int dq q^3 |U|^2 \frac{\text{Im} \chi_1 \text{Im} \chi_2}{\sinh^2(\omega/2T)}$$

$U$  – **screened** inter-layer potential.  $\chi$  – Density response function (diffusive FL)

# Vortex drag in thin films: Results

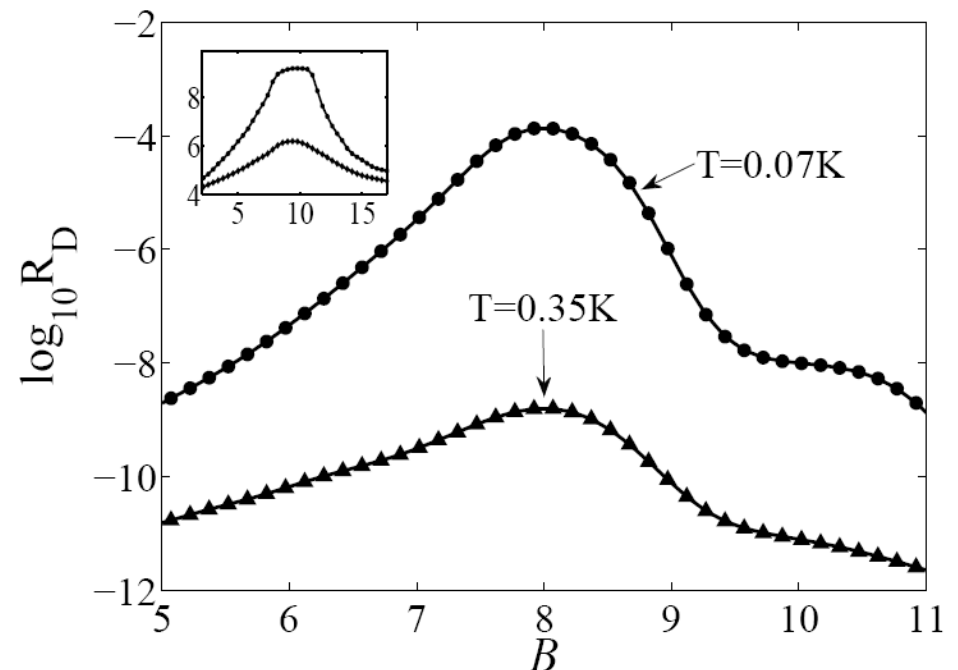
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- Our best chance (with no J tunneling) is the highly insulating InO:



- maximum drag:

$$R_D^{\max} \sim 0.1 m\Omega$$



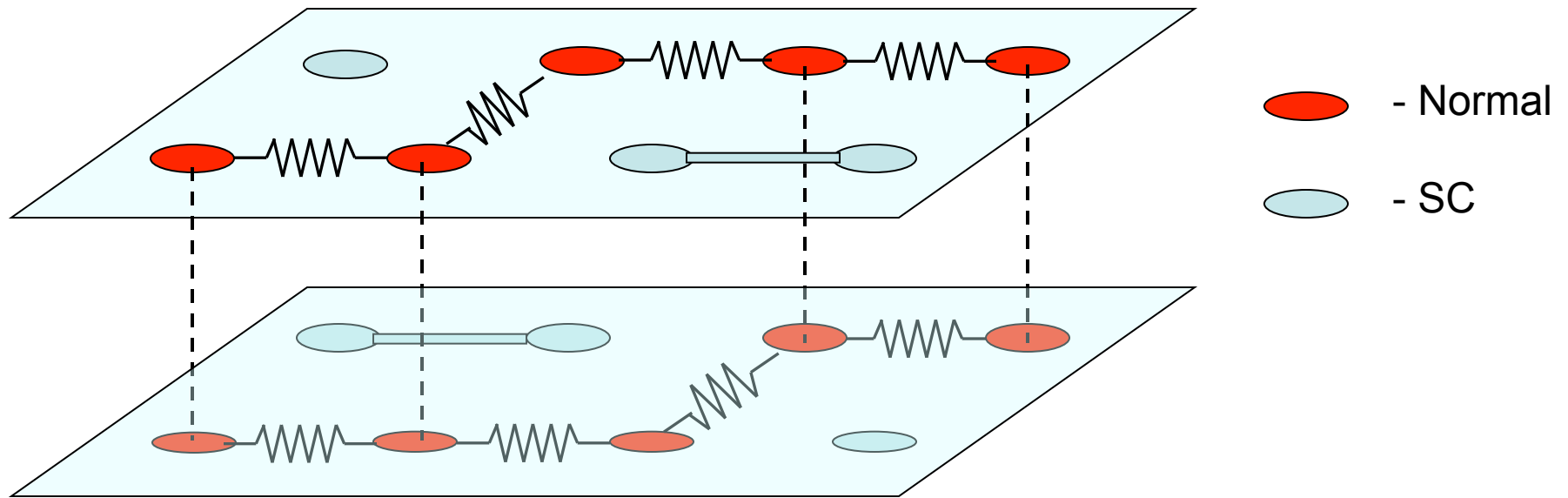
**Note:** similar analysis for SC-metal ‘bilayer’ using a ground plane.

Experiment: Mason, Kapitulnik (2001)

Theory: Michaeli, Finkel’stein, (2006)

## Percolation picture: Coulomb drag

- Solve a 2-layer resistor network with drag.



- Can neglect drag with the SC islands:

$$R_{SC-SC}^D, R_{SC-NOR}^D \approx 0$$

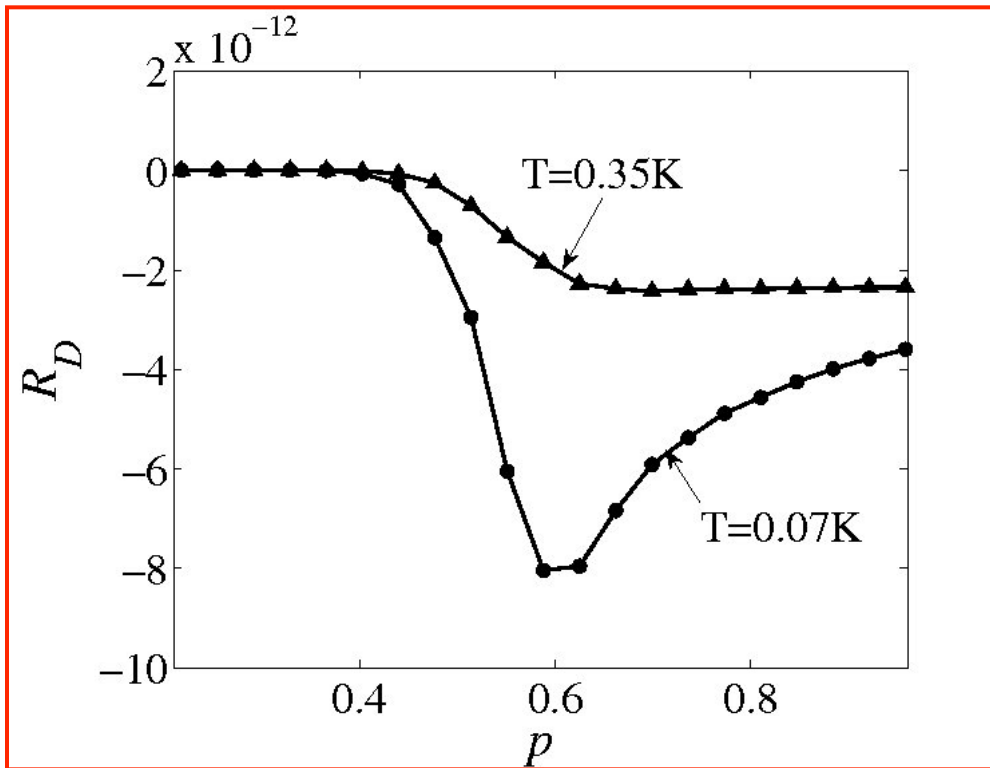
- Normal-Normal drag – use results for disorder localized electron glass:

$$R_{NOR-NOR}^D = \frac{1}{96\pi^2} \frac{R_1 R_2}{\hbar / e^2} \frac{T^2}{(e^2 n_e a d_{film})^2} \ln \frac{1}{2x_0}$$

(Shimshoni, PRB 1994)

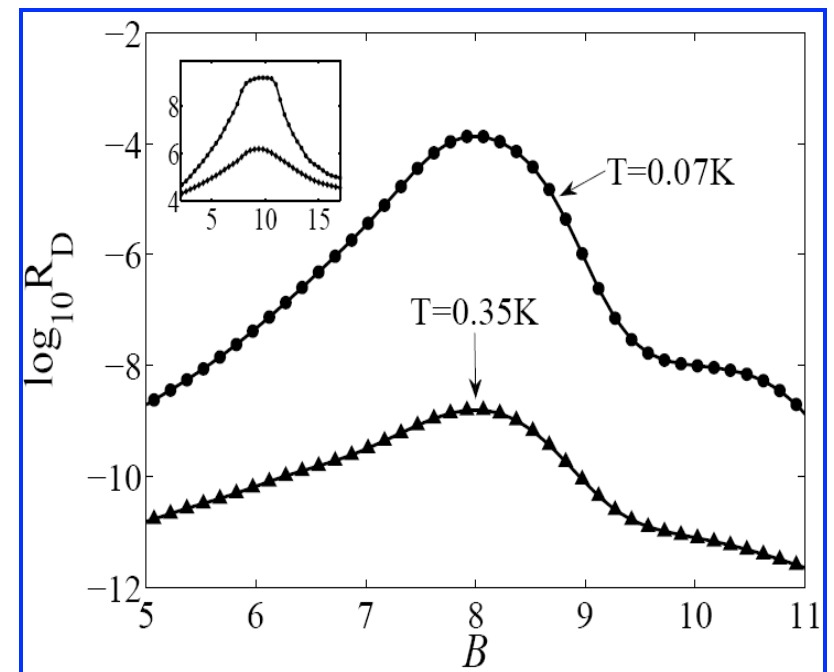
## Percolation picture: Results

- Solution of the random resistor network:



Compare to vortex drag:

?



## Conclusions

- Vortex picture and the puddle picture: similar single layer predictions.
- Giaever transformer bilayer geometry may qualitatively distinguish:  
Large drag for vortices, small drag for electrons, with opposite signs.

- Drag in the limit of zero interlayer tunneling:

$$R_D^{vortex} \sim 0.1m\Omega \quad \text{vs.} \quad R_D^{percolation} \sim 10^{-11}\Omega$$

- Intelayer Josephson should increase both values, and enhance the effect.  
(future theoretical work)
- Amorphous thin-film bilayers will yield interesting complementary information about the SIT.

## Conclusions

- What induces the gigantic resistance and the SC-insulating transition?
- What is the nature of the insulating state? Exotic vortex physics?

### Phenomenology:

- Vortex picture and the puddle picture: similar single layer predictions.

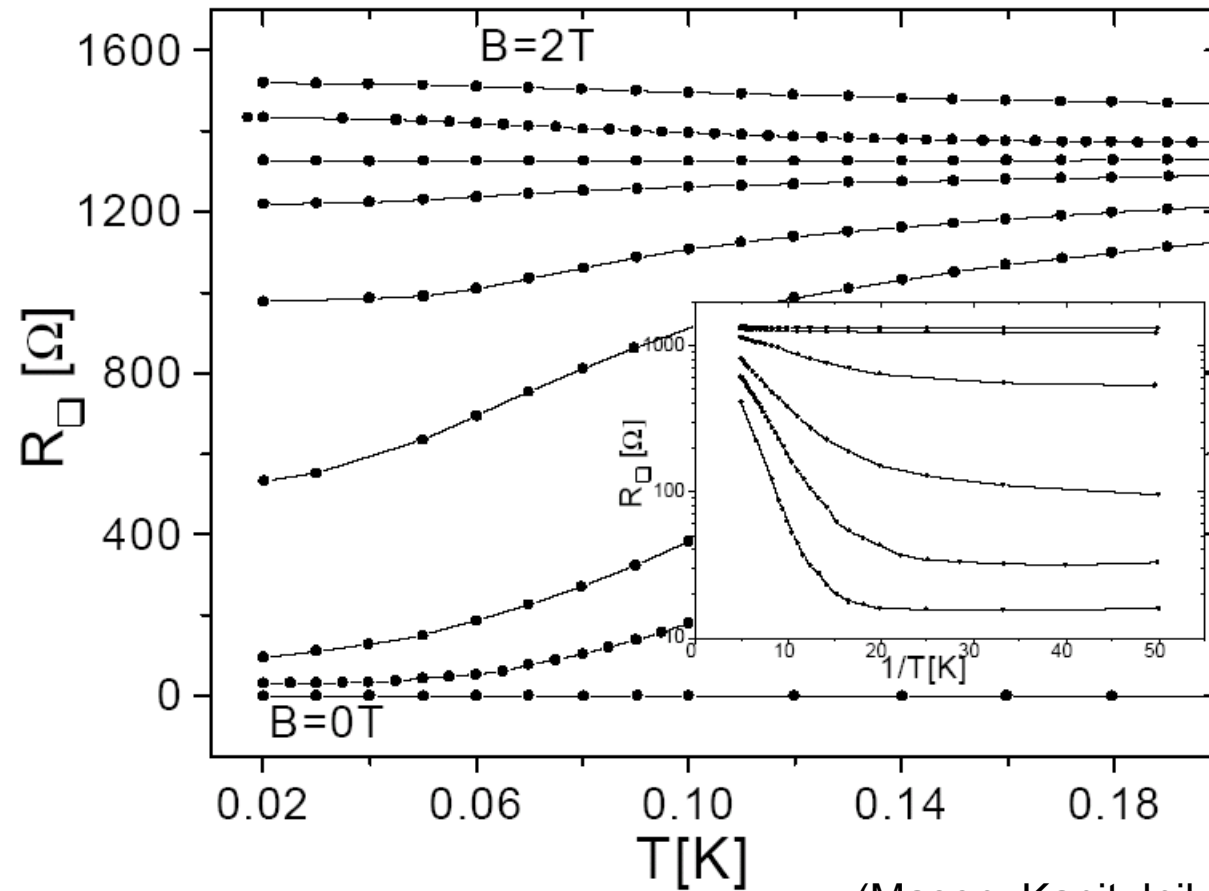
### Experimental suggestion:

- Giaever transformer bilayer geometry may qualitatively distinguish:  
Large drag for vortices, small drag for electrons.



## Observation of a metallic phase

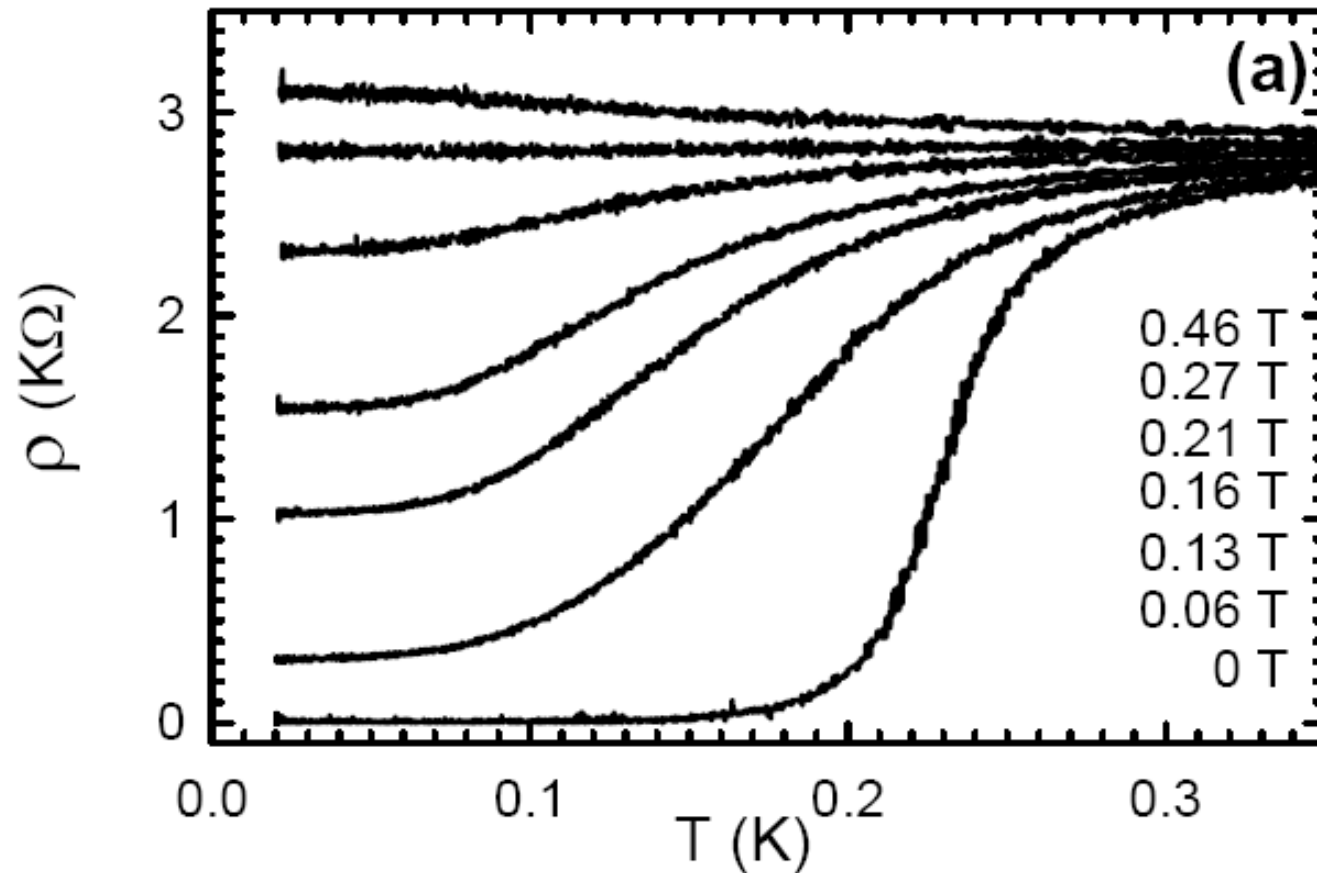
- MoGe:



(Mason, Kapitulnik, PRB 1999)

## Observation of a metallic phase

- Ta:



(Qin, Vicente, Yoon, 2006)

- Saturation at  $\sim 100\text{mK}$ : New metallic phase? (or saturation of electrons temperature)

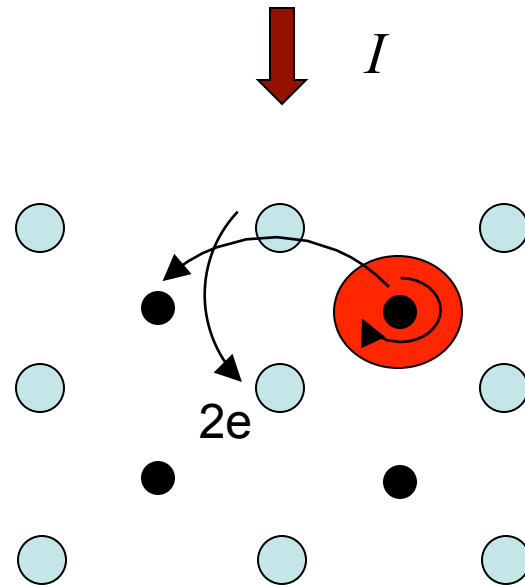
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- EMF due to vortex hopping:

$$\frac{\hbar}{2e} \Delta \dot{\phi} = \Delta V \quad \rightarrow \quad \Delta V = \frac{\hbar}{2e} \frac{2\pi}{\tau}$$

- Resistance:  $R = \frac{V}{I} = \frac{2\pi\hbar}{2e\tau} \bigg/ \frac{2e}{\tau} = \frac{h}{4e^2} = 6.5k\Omega$



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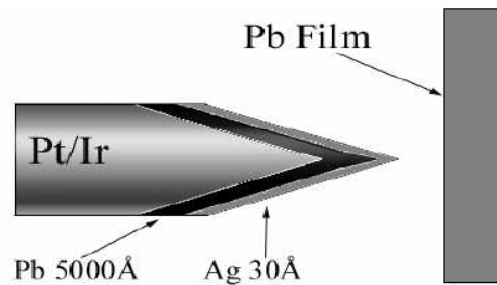
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## More physical properties of the vortex metal

### *Cooper pair tunneling*

- A superconducting STM can tunnel Cooper pairs to the film:

$$G = G_{2e} + G_{CP}$$



(Naaman, Tyzer, Dynes, 2001).

# More physical properties of the vortex metal

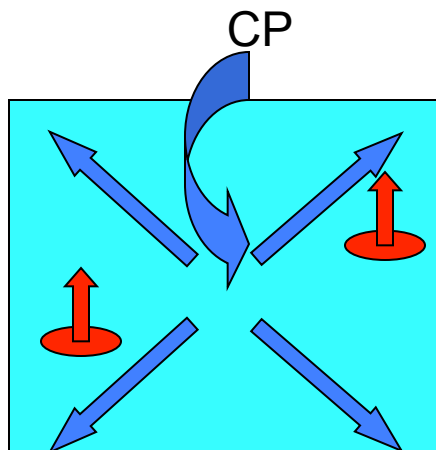
## Cooper pair tunneling

- A superconducting STM can tunnel Cooper pairs to the film:

$$G = G_{2e} + G_{CP}$$

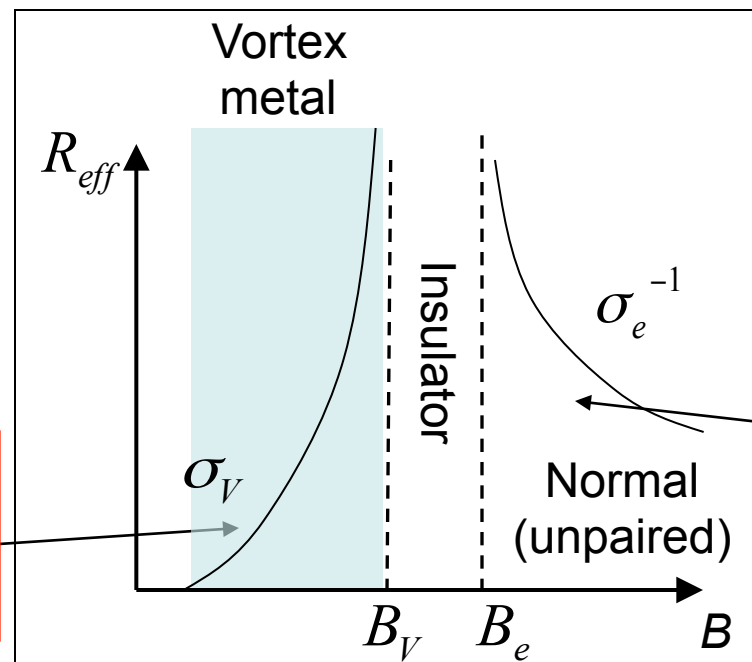
Vortex metal phase:

$$G_{CP} \sim \frac{1}{T^2} \exp(-\sigma_V \ln^2 T)$$



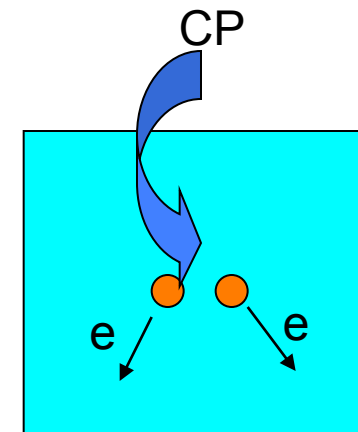
$$G \approx G_{CP}$$

strongly T dependent



Normal phase:

$$G_{2e} \sim \sigma_e^2$$



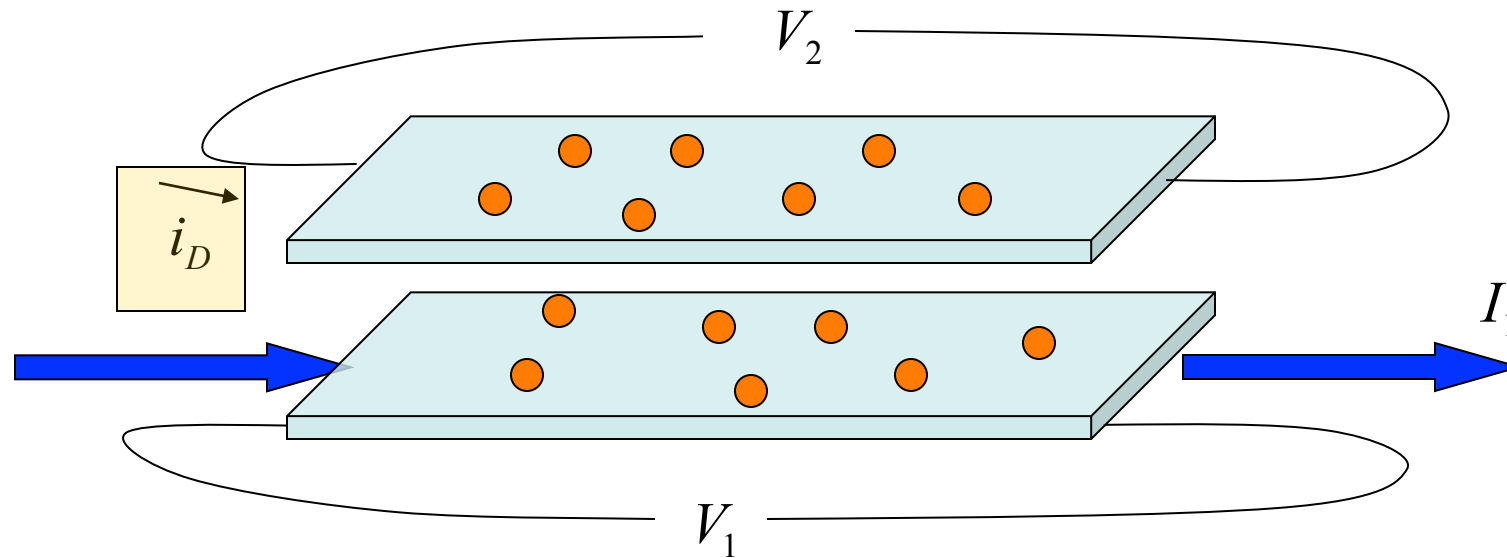
$$G \approx G_{2e}$$

$$\sim 1/\ln^2 T$$

## 2DEG bilayers – Coulomb drag

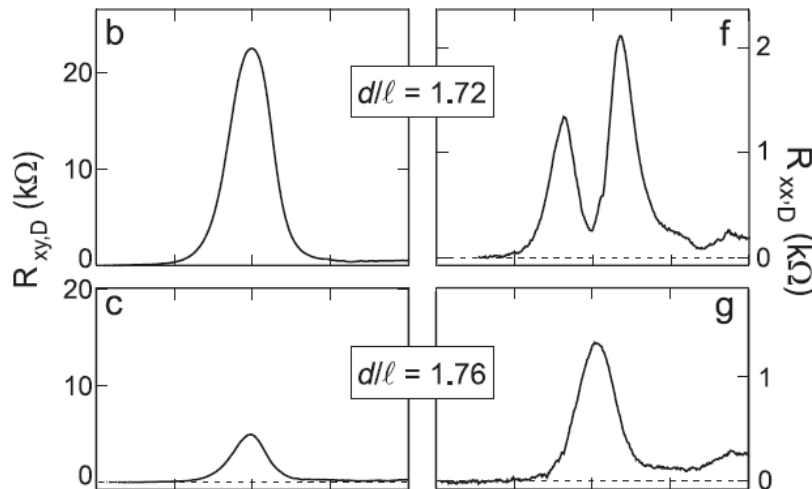
Two thin electron gases:

$$R_D = \frac{V_2}{I_1}$$



Example:

$\nu_T = 1$  “Excitonic condensate”



(Kellogg, Eisenstein, Pfeiffer, West, 2002)